Stats 2 - January 2006

① $X \sim Po(1.5)$

a) i) $P(X=2) = \dfrac{e^{-1.5} \times (1.5)^2}{2!} = 0.251$ (3dp)

ii) $P(X=2)^3 = 0.251^3 = 0.0158$

b) i) $Y \sim Po(9)$ $[6 \times 1.5]$

ii) $P(Y \geq 12) = 1 - P(Y \leq 11) = 1 - 0.8030 = 0.197$

c) The bug attacks patients randomly and independently

② a) H_0: choice of sport is independent of gender

H_1: choice of sport is affected by gender (non-independent)

Observed

	Squash	Badminton	Archery	Hockey	
Male	5	16	30	19	70
Female	4	20	33	53	110
	9	36	63	72	180

Expected

	Squash	Badminton	Archery	Hockey
Male	3.5	14	24.5	28
Female	5.5	22	38.5	44

$\dfrac{\text{Row} \times \text{Collum}}{\text{Total}}$

As expected value for squash < 5, must combine

Combine Squash and Badminton as similar categories:

Expected

	Squash + Bad	Archery	Hockey
Male	17.5	24.5	28
Female	27.5	38.5	44

$\dfrac{(O-E)^2}{E}$

X^2 Values

	SB	A	H
M	0.7	1.2347	2.8928
F	0.4455	0.7857	1.8409

Sum of X^2 values : 7.90 = Test Statistic

Degrees of freedom (v) : $(3-1) \times (2-1) = 2$

Critical Value : $X^2_{5\%}(2) = 5.991$

Reject H_0 as $7.90 > 5.991$

There is enough evidence at 5% level of significance to suggest an association between sport and gender

b) More females and fewer males choose hockey than would be expected if choices independent of gender.

③ a) Don't know population variance, so use t-distribution

From calc : $n = 9$ $\quad \bar{x} = 8$

$\Sigma x = 72$ $\quad s = 2.1213$

$\Sigma x^2 = 612$ $\quad s^2 = 4.5$

Degrees of freedom $= n - 1 = 8$

t value for 90% = 1.860 (look up 0.95)

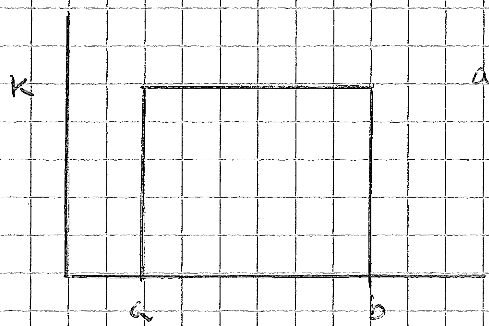
Confidence interval $= 8 \pm 1.86 \times \left(\dfrac{2.1213}{\sqrt{9}}\right)$ $\quad \dfrac{s}{\sqrt{n}}$

$= 8 \pm 1.315$

$= (6.68,\ 9.32)$

b) Head teachers' claim not supported by evidence at 10% level as 5 lies outside of the confidence interval

④



a) i) $K \times (b-a) = 1 \rightarrow K = 1/b-a$

ii) $E(X) = \int x\, f(x)\, dx$

$= \int_a^b xK\, dx$

$= \left[\dfrac{x^2}{2} K \right]_a^b$

K = just a number

$$= \left[\left(\frac{x^2}{2}\right)\left(\frac{1}{b-a}\right) \right]_a^b$$

$$= \left(\frac{1}{2(b-a)}\right) \left[x^2 \right]_a^b$$

$$= \frac{1}{2(b-a)} \left[b^2 - a^2 \right]$$

$$= \frac{1}{2(b-a)} \left((b+a)(b-a) \right)$$

$$= \tfrac{1}{2}(a+b)$$

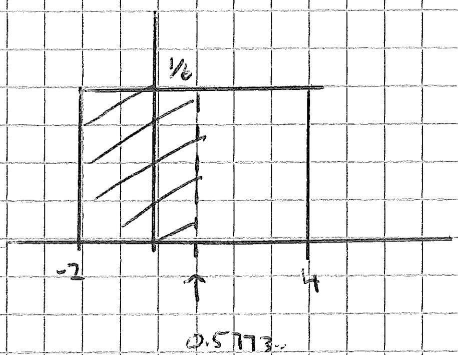
b) i) $\mu = \tfrac{1}{2}(-2+4) = 1$

ii) For rectangular distribution $\sigma^2 = \tfrac{1}{12}(b-a)^2$

$$= \tfrac{1}{12}(4+2)^2 = 3$$

$\therefore \sigma = \sqrt{3}$

iii) $P\left(X < \dfrac{2-\mu}{\sigma}\right) = P\left(X < \dfrac{1}{\sqrt{3}}\right) = P(X < 0.5773.)$



$= \tfrac{1}{6} \times (2 + 0.5773.)$

$= 0.430$

⑤ a) Mean $= 40 \times 0.3 + 45 \times 0.24 + 55 \times 0.36 + 74 \times 0.1$

$= 50 = E(X)$

$E(X^2) = 40^2 \times 0.3 + 45^2 \times 0.24 + 55^2 \times 0.36 + 74^2 \times 0.1$

$= 2602.6$

$Var(X) = E(X^2) - E(X)^2$

$= 2602.6 - 50^2 = 102.6$

$\therefore sd(X) = \sqrt{102.6} = 10.129...$

b) Mean $= 10 \times 50 + 250 = 750$

SD $= 10 \times 10.129.. = 101.29...$

6) a) $H_0: \mu = 65$

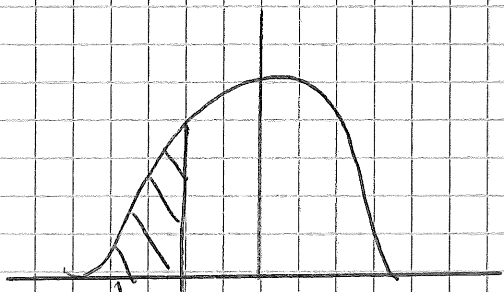
$H_1: \mu < 65$ (1TT)

{we know population ... so use Z}

If H_0 true: $\bar{X} \sim N(65, 9^2/35)$

Test Statistic: $Z = \dfrac{61.5 - 65}{9/\sqrt{35}} = -2.30$

Critical value: 5%, 1 tailed test, $= -1.6449$



$-2.30 < -1.6449$

Reject H_0

Evidence at 5% level suggests students may be under-achieving

b) Type I error = Reject H_0 when True

→ Conclude students are underachieving when they are not

7) a) $E(T) = \int t f(t) = \int_0^1 t \cdot 4t(1-t^2)\ dt$

$= \int_0^1 4t^2 - 4t^4\ dt$

$= \left[\frac{4}{3}t^3 - \frac{4}{5}t^5\right]_0^1$

$= \frac{4}{3} - \frac{4}{5} - 0 = \frac{8}{15}$

b) i) $F(t) = P(T \le t)$

$= \int_0^t f(t) = \int_0^t 4t(1-t^2)\ dt$

$= \int_0^t 4t - 4t^3\ dt$

$= \left[2t^2 - t^4\right]_0^t = 2t^2 - t^4$

ii) $P(\mu < T < \text{med}) = P(T < \text{med}) - P(T < \text{mean})$

$= F(\text{med}) - F(\mu)$ using cum distribution

F(med) always 0.5

$= \cancel{F(\text{med})}\ 0.5 - 2(8/15)^2 - (8/15)^4$

$= 0.5 - 0.4879\ldots$

$= 0.0120\ldots$

(8) $H_0: \mu = 1000$

$H_1: \mu \neq 1000$ (2 tailed)

We don't know population SD, so need t-distribution

From calc:

$n = 12$ $\quad\quad \bar{x} = 1003$

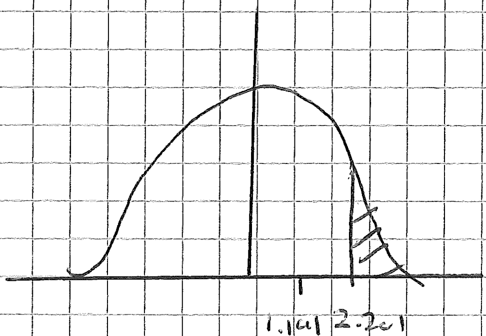
$\Sigma x = 12036$ $\quad\quad s = 5.4439..$

$\Sigma x^2 = 12072434$ $\quad\quad s^2 = 29.636..$

Test statistic $= \dfrac{1003 - 1000}{5.4439/\sqrt{12}} = 1.91$

Critical Value: $\nu = 12 - 1 = 11$

2 Tailed: $t_{5\%}(11) = \pm 2.201$



Accept H_0

Not enough evidence at 5% level to suggest a change in the mean content of sherry in a bottle